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epsilon not

Example # 1.

In figure (a) two positively charged particles are fixed in place on x-axis. The charges are $q_1 = 1.60 \times 10^{-19} C$ and $q_2 = 3.20 \times 10^{-19} C$. and the particle separation is $R = 0.0200\, m$. what are the magnitude and direction of electrostatic force $\vec{F}_{12}$ on particle 1. from particle 2?

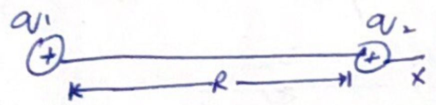


fig (a)

Solution.

Because both particles are positively charged so they are repelled by each other.

The magnitude of this force is

$$F_{12} = \frac{1}{4\pi\varepsilon_0} \frac{|q_1||q_2|}{R^2}$$

$$= (8.99 \times 10^9 N.m^2/C^2) \times \frac{(1.60 \times 10^{-19} C)(3.20 \times 10^{-19} C)}{(0.0200 m)^2}$$

$$= 1.15 \times 10^{-24} N$$

This force has the following magnitude and direction

$$\vec{F}_{12} = 1.15 \times 10^{-24} N \text{ and } 180^\circ$$

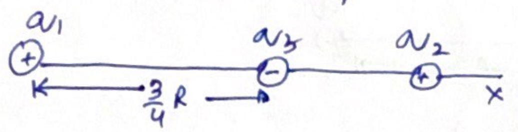
In unit vector notation.

$$\vec{F}_{12} = -(1.15 \times 10^{-24} N)\hat{i}$$

∴ -ve due to opposite direction.

Example # 2.

figure (b) is identical to figure (a) except that particle 3 lies b/w particle 1 and 2. particle 3 has charge $q_3 = -3.20 \times 10^{-19}$ C and is at a distance $\frac{3}{4}R$ from particle 1. what is net electrostatic force $\vec{F}_{net}$ on particle 1. due to particle 2 and 3.



Solution.

To find the magnitude $\vec{F_{13}}$

$$F_{13} = \frac{1}{4\pi\varepsilon_0} \frac{|q_1||q_3|}{(\frac{3}{4}R)^2}$$

$$= (8.99 \times 10^9 \, N.m^2/C^2) \times \frac{(1.60 \times 10^{-19} C)(3.2 \times 10^{-19} C)}{(\frac{3}{4})^2 (0.0200 \, m^2)}$$

$$= 2.05 \times 10^{-24} N.$$

In unit vector notation.

$$\vec{F}_{13} = (2.05 \times 10^{-24} N)\,\hat{i}.$$

The net force $\vec{F}_{1,Net}$ on particle 1

$$\vec{F}_{1net} = \vec{F}_{12} + \vec{F}_{13}$$

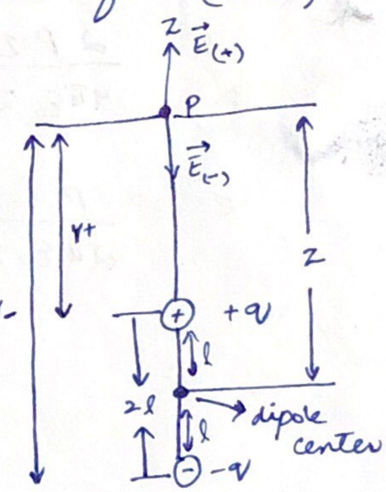
$$= -(1.15 \times 10^{-24} N)\hat{i} + (2.05 \times 10^{-24} N)\hat{i}$$

$$= (9.00 \times 10^{-25} N)\,\hat{i}.$$

The electric field due to an electric dipole.

Two particles that have the same charge magnitude q but opposite signs, a very common and important arrangement known as electric dipole. The particles are separated by distance $2l$. and lie along the dipole axis. Suppose ~~the~~ z as a dipole axis.

Figure (1) shows electric fields sets up at P by each particle. The nearer particle with charge $+q$ sets up field $E_{(+)}$ in the positive direction of the z axis. The other particle with the charge $(-q)$ sets up a smaller field (E_-) in the negative direction.



Electric field at P due to $+q$, $E = +\frac{1}{4\pi\varepsilon_0}\frac{q}{(z-l)^2}$

" " " $-q$, $E = -\frac{1}{4\pi\varepsilon_0}\left(\frac{q}{(z+l)^2}\right)$

The magnitude of net field at P is

$$E = E_{(+)} - E_{(-)}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2_{(+)}} - \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2_{(-)}}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{(z-l)^2} - \frac{1}{4\pi\varepsilon_0}\frac{q}{(z+l)^2}$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{(z-l)^2} - \frac{1}{(z+l)^2}\right)$$

$$E = \frac{q}{4\pi\varepsilon_0}\left(\frac{(z+l)^2-(z-l)^2}{(z^2-l^2)^2}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\,\frac{4zl}{(z^2-l^2)^2}$$

$$= \frac{2Pz}{4\pi\varepsilon_0(z^2-l^2)^2}$$

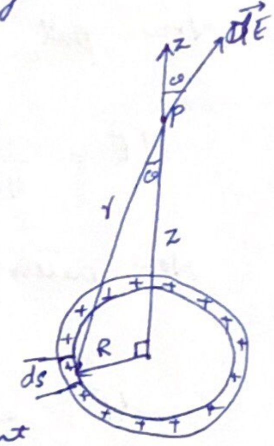
$P = 2l.q$ = dipole moment.

The compare to any distance of z l is very small quantity. Thus in our approximation we can neglect the term l because $l << 1$.

$$E = \frac{2Pz}{4\pi\varepsilon_0 z^4}$$

$$E = \frac{P}{2\pi\varepsilon_0 z^3} \quad \text{(electric dipole)}$$

→ Electric field due to ring of charge.



Let's find the field along the z axis only. Ring has radius R, charge per unit length λ. By symmetry, only E_z is non-zero (the x-y component cancels). A differential element of charge occupies a length ds. This element sets up an electric field $d\vec{E}$ at point P.

The linear charge density is

$$\lambda = \frac{dq}{ds}$$

~~So~~ where

$$dq = \lambda ds.$$

Electric field due to z component is

$$dE = dE_z \cos\theta \quad \text{———} \quad (1)$$

Electric field due to a point charge is

$$dE_z = \frac{1}{4\pi\varepsilon_0} \frac{dq}{r^2} \quad \text{———} \quad (2)$$

Now, we can replace the $\cos\theta$ with legal symbol by using right triangle

$$\cos\theta = \frac{z}{r} \quad \text{———} \quad (3)$$

where

$$r^2 = z^2 + R^2$$

and

$$r = \sqrt{z^2 + R^2}$$

Now put equations (2) and (3) in equation (1)

$$dE = \frac{1}{4\pi\varepsilon_0} \frac{dq}{r^2} \frac{z}{r} \quad \text{——} (4)$$

Now putting values of r, r^2 and dq in equation (4)

$$dE = \frac{1}{4\pi\varepsilon_0} \frac{\lambda ds}{z^2+R^2} \frac{z}{(\sqrt{z^2+R^2})}$$

$$dE = \frac{1}{4\pi\varepsilon_0} \frac{\lambda ds\, z}{(z^2+R^2)^{3/2}}$$

Because we must sum a huge number of these components, each small, we set up an integral that moves along the ring, through the full circumference ($S = 2\pi R$)

$$\int dE = \int \frac{1}{4\pi\varepsilon_0} \frac{\lambda ds\, z}{(z^2+R^2)^{3/2}}$$

$$E = \frac{1}{4\pi\varepsilon_0} \frac{\lambda z}{(z^2+R^2)^{3/2}} \int ds \qquad \therefore \int ds = 2\pi R$$

$$E = \frac{1}{4\pi\varepsilon_0} \frac{\lambda z}{(z^2+R^2)^{3/2}} 2\pi R. \quad \text{——} (5)$$

Now we can switch to the total charge by using

$\lambda = q/2\pi R$

where

$q = \lambda 2\pi R$

Putting ~~value~~ ~~of~~ q in equation (5).

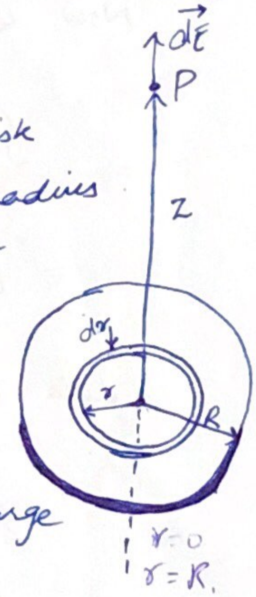
$$E = \frac{1}{4\pi\varepsilon_0} \frac{zq}{(R^2+z^2)^{3/2}} \quad \text{——} (6)$$

Let us suppose the point on the central axis is so far that $z >> R$ then the expression $z^2 + R^2$ ~~can~~ can be approximated as z^2 and the equation 6 becomes.

$$E = \frac{1}{4\pi\varepsilon_0} \frac{zq}{z^3} = \frac{1}{4\pi\varepsilon_0} \frac{q}{z^2}$$

→ Electric field due to disk of charge

For a two dimensional object like a uniformly charged disk, we define a surface charge density σ (charge per unit area) and consider the field at point P on the central axis at the distance z from the center of axis.



we superimpose a ring on the disk as shown in figure, at an arbitrary radius $r \leq R$. The ring is so thin that we can treat the charge on it as a charge element dq. To find its smallest contribution $d\vec{E}$ to the field at point P, we write the equation in terms of the ring's charge dq and radius r.

$$dE = \frac{dq\, z}{4\pi\varepsilon_0 (z^2+r^2)^{3/2}} \quad \text{——(1)}$$

To find the total field at point P. we integrate equation (1) from the center of disk at $r=0$ out to the $r=R$. so that we sum all the dE contributions.

Because the ring is so thin having a thickness dr. Then the surface area dA is the product of circumference $2\pi r$ and its thickness dr.

So in terms of surface charge density σ we have

$$dq = \sigma dA = \sigma(2\pi r dr) \qquad \sigma = \frac{Q}{A}$$

Putting the value of dq in equation (1)

$$dE = \frac{\sigma 2\cancel{\pi} r\, dr\, z}{4\cancel{\pi}\varepsilon_0 (z^2+r^2)^{3/2}}$$

Now taking the integral

$$\int dE = \int_0^R \frac{\sigma\, 2r\, dr\, z}{4\varepsilon_0 (z^2+r^2)^{3/2}}$$

$$E = \frac{\sigma z}{4\varepsilon_0}\int_0^R (z^2+r^2)^{-3/2}\, 2r\, dr$$

Lets suppose that

$$x = z^2 + r^2$$

$$\frac{dx}{dr} = 2r$$

$$dx = 2r\, dr$$

So

$$E = \frac{\sigma z}{4\varepsilon_0}\int x^{-3/2}\, dx$$

$$= \frac{\sigma z}{4\varepsilon_0}\left(\frac{x^{-1/2}}{-1/2}\right) = -\frac{2\sigma z}{4\varepsilon_0}\left(\frac{1}{\sqrt{x}}\right)$$

putting the values of x and taking limits

$$= -\frac{2\sigma z}{4\varepsilon_0}\left(\frac{1}{\sqrt{z^2+R^2}}\Bigg|_0^R\right)$$

$$E = -\frac{\cancel{2}\sigma z}{\underset{2}{\cancel{4}}\varepsilon_0}\left(\frac{1}{\sqrt{R^2+z^2}} - \frac{1}{\sqrt{z^2}}\right)$$

$$= \frac{\sigma z}{2\varepsilon_0}\left(\frac{1}{z} - \frac{1}{\sqrt{R^2+z^2}}\right)$$

$$= \frac{\sigma}{2\varepsilon_0}\left(\frac{z}{z} - \frac{z}{\sqrt{R^2+z^2}}\right)$$

$$= \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{R^2+z^2}}\right)$$ charged disk.